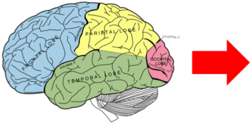
# Epistemic Game Theory: Language and Observation

Adam Brandenburger

NYU Stern School of Business
NYU Polytechnic School of Engineering
NYU Shanghai

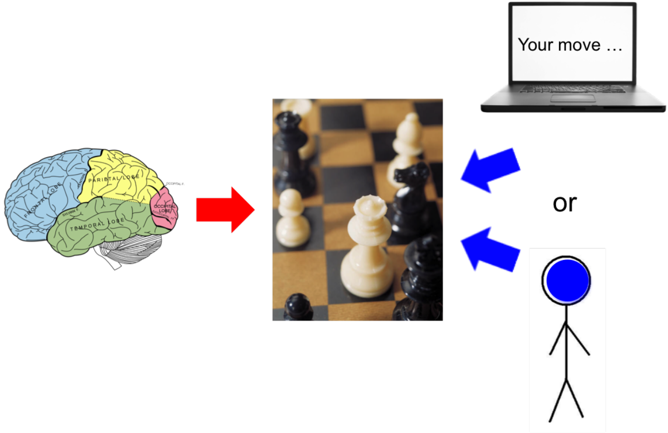
October 4, 2015

# "Theory of Mind" in Tasks




Gallagher, H. et al.,"Reading the Mind in Cartoons and Stories: An fMRI Study of 'Theory of Mind' in Verbal and Nonverbal Tasks," *Neuropsychologia*, 38, 2000, 11-21
http://upload.wikimedia.org/wikipedia/commons/thumb/1/1a/Gray728.svg/1024px-Gray728.svg.png

# “Theory of Mind” in Games




McCabe, K. et al.,“A Functional Imaging Study of Cooperation in Two-Person Reciprocal Exchange,” *PNAS*, 98, 2001, 11832-11835
Gallagher, H., et al., “Imaging the Intentional Stance in a Competitive Game,” *NeuroImage*, 16, 2002, 814-821
Rilling, J., et al., “The Neural Correlates of Theory of Mind Within Interpersonal Interactions,” *NeuroImage*, 22, 2004, 1694-1703
http://upload.wikimedia.org/wikipedia/commons/thumb/1/1a/Gray728.svg/1024px-Gray728.svg.png

# Ellsberg Uncertainty

# Ellsberg Uncertainty

*"These particular uncertainties — as to the other player's beliefs about oneself — are almost universal, and it would constrict the application of a game theory fatally to rule them out"*

– Daniel Ellsberg (1959)

# From Theoretical Impossibilities to Cognitive Possibilities

How many levels of reasoning do players in a game undertake?

Why not posit infinite levels of reasoning and approximate reality this way?

Leaving aside whether we should, we may simply be unable to!

Infinite levels of reasoning can lead to an impossibility result (Brandenburger-Friedenberg-Keisler 2008; but see also Lee 2015)

自相矛盾

# The Epistemic Language

Fix a game (matrix or tree)

An associated **epistemic game** adds sets $T_a$ and $T_b$ of **epistemic types** for Ann and Bob respectively, and maps:

$$\begin{aligned} \lambda_a &: T_a \to \mathcal{M}(S_b \times T_b) \\ \lambda_b &: T_b \to \mathcal{M}(S_a \times T_a) \end{aligned}$$

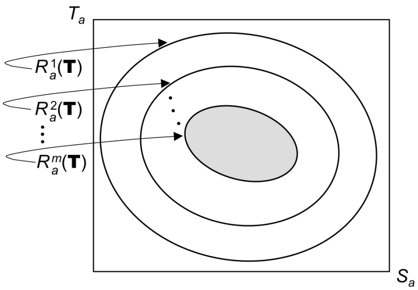
where $\mathcal{M}(\cdot)$ might be the space of probability measures on a $\sigma$-algebra on $S_i \times T_i$, or another belief modality

For a fixed game, different tuples $(T_a, T_b, \lambda_a, \lambda_b)$ yield different epistemic games

# What Does Epistemic Game Theory Look Like?

Fix a game (matrix or tree) $\Gamma$ and add an epistemic type structure:

$$\mathbf{T} = (T_a, T_b, \lambda_a, \lambda_b)$$




- $R_a^1(\mathbf{T})$ is the set of strategy-type pairs for Ann that are rational
- $R_a^2(\mathbf{T})$ is the set of strategy-type pairs for Ann that are rational and think Bob is rational
- ...

# What Do the Words Mean?

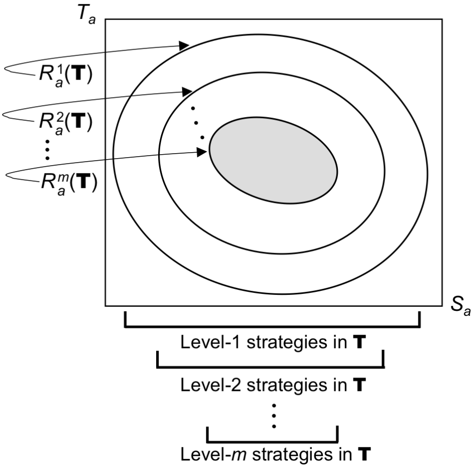
The words "rational" and "think" can be formalized to capture various different meanings

These different choices give rise to different epistemic analyses

But there is a common architecture as indicated by the diagram

The projections of the $R_a^m(\mathbf{T})$ sets yield the strategies consistent with the given epistemic condition

# Level-$m$ Strategies




A strategy $s_a$ is **level-**$m$ if there is a type structure $\mathbf{T}$ such that $s_a \in \mathrm{proj}_{S_a} R_a^m(\mathbf{T})$

# Identifying Level-$m$ Strategies

We could try to search over all type structures

Question: Is there a particular type structure $\mathbf{T}^{\cup}$ such that for all $m$:

$$\mathrm{proj}_{S_a} R_a^m(\mathbf{T}^{\cup}) = \bigcup_{\mathbf{T}} \mathrm{proj}_{S_a} R_a^m(\mathbf{T})$$

We always have "the fundamental inclusion of epistemic game theory":

$$\mathrm{proj}_{S_a} R_a^m(\mathbf{T}^{\cup}) \subseteq \bigcup_{\mathbf{T}} \mathrm{proj}_{S_a} R_a^m(\mathbf{T})$$

Question: Are there cases with equality?

# Baseline Check

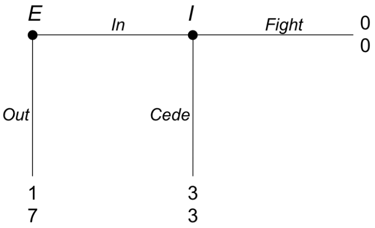
- In the matrix, when "rational" means "maximize subjective expected payoff" and "think" means "believe" (assign probability 1 to), then $\mathbf{T}^{\cup}$ can be taken to be the **canonical** type structure $\mathbf{T}^{*}$ (Mertens and Zamir 1985, Brandenburger and Dekel 1993), and $\mathrm{proj}_{S_a} R_a^m(\mathbf{T}^{\cup})$ coincides with the set of strategies that survive $m$ rounds of elimination of strongly dominated strategies ("rationalizability")

# Some New Findings

- In the tree, when "rational" means "maximize expected payoff under a conditional probability system" and "think" means "strongly believe" (i.e. assign probability 1 to, whenever possible), and if $\mathbf{T}^*$ is the canonical type structure (Battigalli and Siniscalchi 2002), then $\mathrm{proj}_{S_a} R_a^m(\mathbf{T}^*)$ coincides with the set of strategies that survive $m$ rounds of elimination of conditionally dominated strategies ("extensive-form rationalizability")

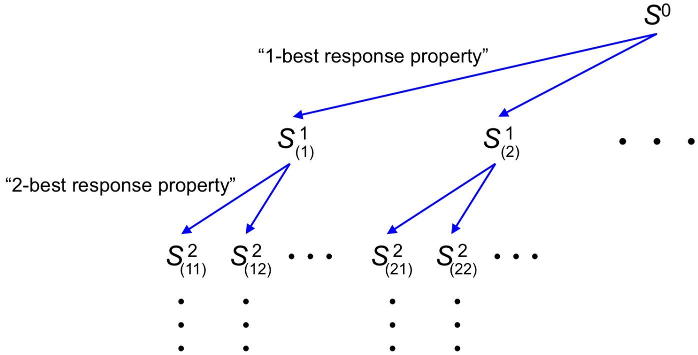
- In the matrix, when "rational means "maximize expected payoff under a lexicographic probability system" and "think" means "assume", and if $\mathbf{T}^*$ is a complete type structure (Brandenburger 2003), then $\mathrm{proj}_{S_a} R_a^m(\mathbf{T}^*)$ coincides with the set of strategies that survive $m$ rounds of elimination of inadmissible strategies (Brandenburger, Friedenberg, and Keisler 2008)

# An Example of Strict Inclusion



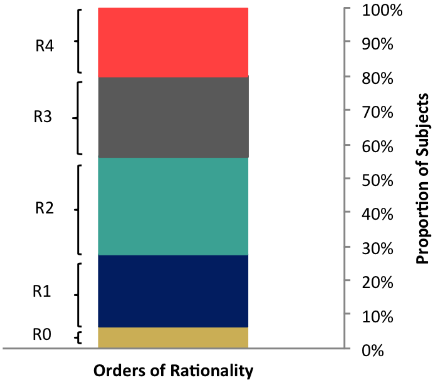

- Consider the twice-repeated Chain-Store game (Selten 1978)
- In the canonical structure $\mathbf{T}^*$ ("context-free"), a level-$m$ entrant (for $m$ sufficiently large) must choose *In* in the first period
- We can build another type structure $\mathbf{T}^!$ ("the incumbent is expected to fight") where a level-$m$ entrant (for all $m$) chooses *Out* in the first period

# Identification of Level-$m$ Strategies in the Tree




- When a set $S^m_{(\cdot\cdot)}$ repeats down a branch, that branch terminates
- All strategies in such $S^m_{(\cdot\cdot)}$ are consistent with $\infty$-level reasoning
- See Brandenburger, Danieli, and Friedenberg (2015) for details

# Empirical Findings in the Matrix




Using a novel ring-game design, Kneeland (forthcoming) found 94 percent of the subjects were level-1, 71 percent were level-2, 44 percent were level-3, 22 percent were level-4, and none were level-5 or higher

# Summary and Outlook

The Kneeland findings results agree broadly with cognition studies (e.g., Stiller and Dunbar 2007)

What experimental results will we find in game trees vs. matrices?

What will happen if we put such data into epistemic game theory and use the resulting models in application areas?

Brandenburger and Li (preliminary draft, 2015) use neural evidence to propose a model of belief formation with complexity which is exponential in the level $m$